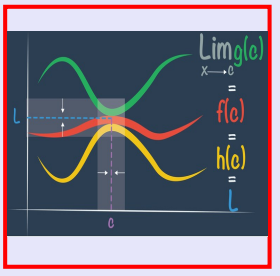
Math 261

Fall 2022

Lecture 40

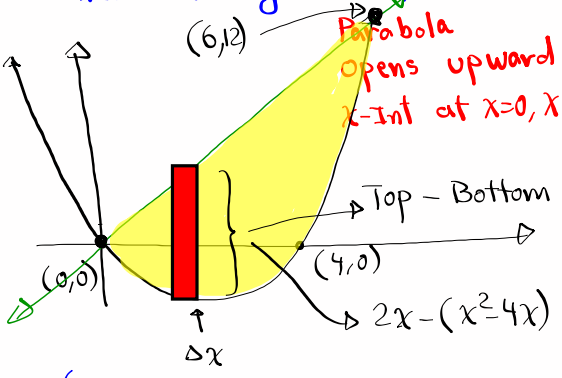


Feb 19-8:47 AM

Find the area enclosed by $f(x)=x^2-4x$ and $g(x)=2x$.

Slant line
x-Int (0,0)

Parabola
Opens upward
x-Int at $x=0, x=4$



$f(x)=g(x)$

$x^2-4x=2x$

$x^2-6x=0$

$x(x-6)=0$

$x=0 \quad x=6$

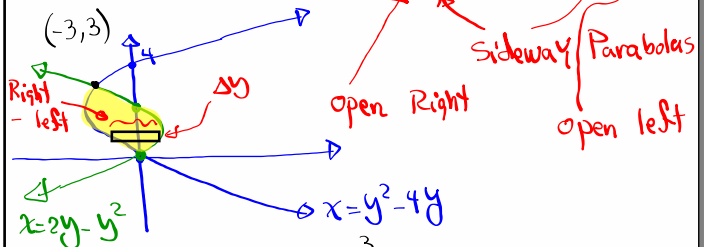
$$A=\int_0^6 [2x-(x^2-4x)]\,dx=\int_0^6 (6x-x^2)\,dx$$

$$=\left(\frac{6x^2}{2}-\frac{x^3}{3}\right)\Big|_0^6$$

$$=\left(3x^2-\frac{1}{3}x^3\right)\Big|_0^6=3(6)^2-\frac{1}{3}(6)^3=\boxed{36}$$

Nov 8-8:46 AM

Find the enclosed Area by $x = y^2 - 4y$ & $x = 2y - y^2$.



Sideway Parabolas: open Right, open left

$2y - y^2 = y^2 - 4y$

$y^2 - 4y - 2y + y^2 = 0$

$2y^2 - 6y = 0$

$2y(y-3) = 0$

$y = 0$ or $y = 3$

$A = \int_0^3 [\text{Right} - \text{left}]\,dy$

$= \int_0^3 [2y - y^2 - (y^2 - 4y)]\,dy$

$= \int_0^3 (6y - 2y^2)\,dy$

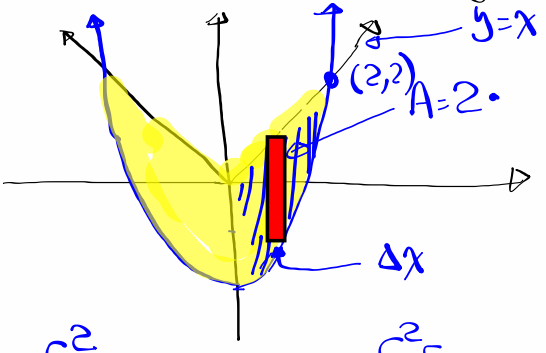
$= (3y^2 - \frac{2}{3}y^3)\Big|_{y=0}^{y=3}$

$= 3 \cdot 3^2 - \frac{2}{3} \cdot 3^3$

$= \boxed{9}$

Nov 8-8:55 AM

Find the area enclosed by $y = |x|$ & $y = x^2 - 2$



$x^2 - 2 = x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$

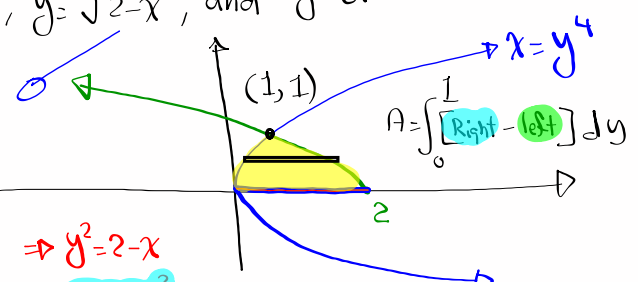
$A = 2\int_0^2 [\text{Top} - \text{Bottom}]\,dx = 2\int_0^2 [x - (x^2 - 2)]\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]\Big|_0^2$

$= 2\left[2 - \frac{8}{3} + 4\right] = \boxed{\frac{20}{3}}$

Nov 8-9:08 AM

Find the area of enclosed region by $x = y^4$, $y = \sqrt{2-x}$, and $y = 0$.



$2 - x \geq 0$

$x \leq 2$

$x = y^4$

$A = \int_0^1 [\text{Right} - \text{left}]\, dy$

$y = \sqrt{2-x} \Rightarrow y^2 = 2 - x$

$x = 2 - y^2$

$x = y^4$

$\Rightarrow y^4 = 2 - y^2 \rightarrow y^4 + y^2 - 2 = 0$

$(y^2+2)(y^2-1) = 0$

has no real Soln.

$y = \pm 1$

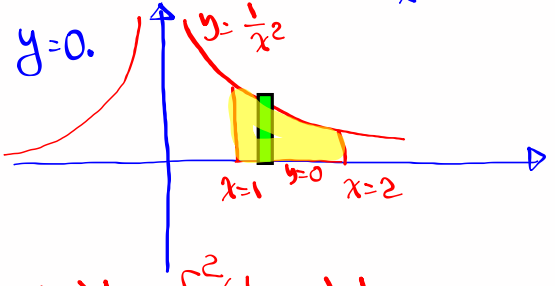
$A = \int_0^1 (2 - y^2 - y^4)\, dy$

$= \left(2y - \frac{y^3}{3} - \frac{y^5}{5}\right)\Big|_0^1$

$= 2 - \frac{1}{3} - \frac{1}{5} = \boxed{\frac{22}{15}}$

Nov 8-9:16 AM

Find the area enclosed by $y = \frac{1}{x^2}$, $x = 1$, $x = 2$, and $y = 0$.



$A = \int_1^2 (\text{Top} - \text{Bottom})\, dx = \int_1^2 \left(\frac{1}{x^2} - 0\right) dx$

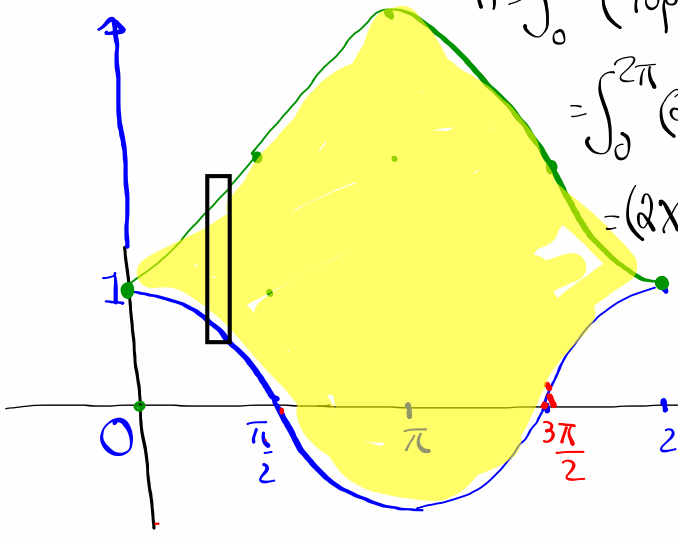
$= \int_1^2 x^{-2}\, dx = \frac{x^{-1}}{-1}\Big|_1^2$

$= -\frac{1}{x}\Big|_1^2$

$= -\left(\frac{1}{2} - 1\right) = \boxed{\frac{1}{2}}$

Nov 8-9:27 AM

Find the area enclosed by $y = \cos x$, $y = 2 - \cos x$, for $0 \le x \le 2\pi$.



$$A = \int_0^{2\pi} (\text{Top} - \text{Bottom})\,dx$$

$$= \int_0^{2\pi} (2 - \cos x - \cos x)\,dx$$

$$= (2x - 2\cdot\sin x)\Big|_0^{2\pi}$$

$$= \boxed{4\pi}$$

Nov 8-9:33 AM

Find $\int (x+2)\,dx = \frac{x^2}{2} + 2x + C$

find $\int (2x - 3)\,dx = x^2 - 3x + C$

$$\int (2x-3)^2\,dx = \int (4x^2 - 12x + 9)\,dx$$

however $= \frac{4x^3}{3} - \frac{12x^2}{2} + 9x + C$

Let $u = 2x - 3$

$du = 2\,dx \rightarrow \frac{du}{2} = dx$

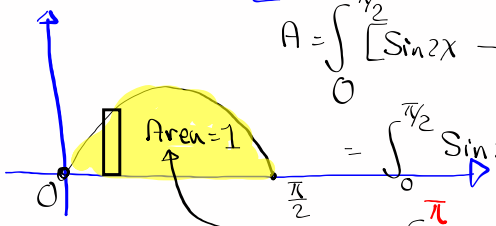
$$\int u^2 \cdot \frac{du}{2} = \frac{1}{2} \cdot \frac{u^3}{3} + C$$

$$= \frac{1}{6}(2x-3)^3 + C$$

Nov 8-9:42 AM

Find the area below $y = \sin 2x$, above $y = 0$ for $0 \le x \le \frac{\pi}{2}$

$\sin 2\left(\frac{\pi}{2}\right) = \sin \pi = 0$



$$A = \int_0^{\pi/2} [\sin 2x - 0]\, dx$$

$$= \int_0^{\pi/2} \sin 2x\, dx$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$x = 0 \to u = 0$

$x = \frac{\pi}{2} \to u = \pi$

$$= \int_0^{\pi} \sin u \cdot \frac{du}{2}$$

$$= \frac{1}{2} \cdot (-\cos u)\Big|_0^{\pi}$$

$$= \frac{-1}{2}\left[\cos \pi - \cos 0\right]$$

$$= \frac{-1}{2}\left[-1 - 1\right]$$

$$= \frac{-1}{2} \cdot (-2) = \boxed{1}$$

Nov 8-9:49 AM